ARCENG
INTERNATIONAL JOURNAL OF ARCHITECTURE AND ENGINEERING

ISSN 2822-6895

Article Arrival Date	Article Type	Article Published Date
18.04.2025	**Research Article**	**20.06.2025**

MATHEMATICAL OPTIMIZATION MODELS USED IN ARTIFICAL INTELLIGENCE ALGORITHMS

BEKİR DANIŞ [1]

[1] Dr., Aydın Adnan Menderes University, Digital Transformation Office,
https://orcid.org/0000-0002-3532-2670

Abstract

In this article, mathematical optimization techniques, which are one of the basic building blocks of artificial intelligence systems, are examined in detail. The place of optimization in artificial intelligence, basic concepts, classical and modern algorithms, application areas and current developments are evaluated. In addition, the role of optimization in the learning processes of artificial intelligence models and the comparison of different optimization methods are discussed. Finally, suggestions and research perspectives for the future of optimization in the field of artificial intelligence are presented.

Keywords: Artifical intelligence, optimization, mathematics

1. INTRODUCTION

Artificial intelligence (AI) has become the intersection of many disciplines today, creating transformative effects in various fields such as health, finance, engineering, transportation, education and defense. One of the most important factors behind this multifaceted domain is the capacity of AI systems to solve complex problems with high accuracy and efficiency. One of the fundamental building blocks that makes this capacity possible is mathematical optimization. Optimization, in its general sense, is the process of finding the most appropriate (minimum or maximum) value of a target function (or objective function) under certain constraints. This process has a central role in both classical deterministic systems and modern data-driven approaches.

In the context of AI models, optimization is often a mechanism that manages the learning process of the model. In machine learning and deep learning algorithms, a loss function is defined that measures the performance of the model. This function measures the difference between the predictions made by the model and the actual values, and the parameters (weights and bias terms) are iteratively updated to minimize this difference. For example, derivative-based optimization algorithms such as gradient descent proceed in small steps to optimize the model parameters by calculating the gradient of the loss function. This process usually involves solving high-dimensional and nonlinear optimization problems.

Optimization is not limited to parameter updates; it is also used in many other decision processes such as the selection of model architecture, hyperparameter adjustments, and data preprocessing strategies. In addition, finding action policies that will maximize the rewards that agents obtain as a result of their interaction with the environment in areas such as reinforcement learning is also an optimization problem in essence.

The problems encountered in modern artificial intelligence applications are usually nonlinear, multivariate, nonconvex, and even discrete optimization problems. Therefore, not only classical optimization techniques, but also stochastic approaches (e.g. Stochastic Gradient Descent), evolutionary algorithms, swarm intelligence-based metaheuristic methods, and algebraic optimization techniques are widely used.

In conclusion, mathematical optimization is one of the fundamental building blocks of artificial intelligence systems and is a component that directly affects the level of success of these systems. It would not be possible for AI to reach its current level of success without the mathematical foundation provided by optimization, both during model training and in the construction and implementation of the model.

2. PRELIMINARIES CONCERNING MATHEMATICAL OPTIMIZATION

Optimization is the process of systematically searching for the best solution for a specific purpose, and this process is based on several basic concepts. First of all, the objective function is the mathematical expression to be optimized; the minimum or maximum of this function is sought. Decision variables are variables that affect the value of the objective function and need to be optimized. Constraints are inequalities or equivalences that the solution must comply with and narrow the solution space. In unconstrained optimization, only the objective function is considered, while in constrained optimization, both the objective function and the constraints are evaluated together. In addition, while the global optimum represents the best solution in the entire solution space, the local optimum represents the best solution only in a specific environment. Convexity is a critical structural feature that affects the uniqueness of the solution in optimization problems and the success of the algorithms. These basic concepts play a central role in modeling, analyzing and solving optimization problems.

Optimization problems focus on finding the maximum or minimum value of an objective function. This function usually expresses the accuracy or cost of the model. Constraints specify the constraints that the solution must obey; they can be linear, nonlinear, equal, or inequality. To clarify this, consider the following cases.

Linear Optimization: Problems where the target function and constraints are linear. Nonlinear Optimization: Target function or constraints are not linear. Integer Optimization: Cases where variables take integer values. Unconstrained Optimization: Cases without constraints. Convex and Nonconvex Optimization: Problem classification according to the structure of the solution set.

3. OPTIMIZATION IN ARTIFICAL INTELLIGENCE MODELS

In AI systems, optimization plays a critical role in tuning model parameters, managing the learning process, and improving overall performance. Learning algorithms rely on optimization techniques to find parameters that minimize model error. Especially in machine learning and deep learning, optimization algorithms are at the core of the training process.

3.1. Optimization Models

Optimization plays a pivotal role in artificial intelligence (AI) systems by enabling the fine-tuning of model parameters, guiding the learning process, and enhancing overall system performance. The success of these systems largely depends on choosing the right parameters and implementing an appropriate learning strategy. In particular, in machine learning (ML) and deep learning (DL) models, millions of parameters are optimized during the training of complex artificial neural networks. In this process, learning algorithms usually try to minimize a loss function; this function measures the difference between the model's predictions and the actual values.

Optimization algorithms are used to find parameter combinations that minimize this loss function. One of the most common methods, gradient descent, and its derivatives (e.g., Stochastic Gradient Descent – SGD, Adam, RMSProp) are basic tools in this context. These algorithms aim to reach the minimum value of the loss function by iteratively updating the model parameters. However, multidimensional, non-convex optimization problems, especially those that arise in deep networks, can make it difficult to reach the global minimum. For this reason, strategies such as learning rate, momentum, and regularization are put into action to ensure that the optimization process progresses efficiently and in a balanced manner.

In addition, optimization is not limited to model training; it is also used in many stages such as hyperparameter selection, design of artificial neural network architectures, feature selection, and even automatic machine learning (AutoML) processes. In this context, higher-level optimization techniques such as heuristic and meta-heuristic methods (e.g. genetic algorithms, grid search, Bayesian optimization) are also preferred.

As a result, optimization algorithms increase both the effectiveness of the learning process of AI systems and the generalization success of the obtained models. Therefore, the selection of powerful and appropriate optimization techniques is a decisive factor in the success of artificial intelligence applications.

3.2. Modern and Meta-Heuristic Optimization Techniques

Many engineering, economic, artificial intelligence and data analytics problems that need to be solved today have complex, multidimensional and nonlinear structures. The solution of such problems with classical mathematical methods is either very difficult or practically impossible. For this reason, modern and meta-heuristic optimization techniques have attracted great attention in recent years.

Modern optimization goes beyond classical methods and offers more flexible and effective solutions to difficulties such as uncertainty, multiple solutions, dealing with constraints and working with high dimensions. These techniques are often inspired by nature, biology or social behavior.

Metaheuristic optimization methods stand out with their ability to intelligently explore the solution space. These methods are not specific to a particular problem structure, but they offer general-purpose solution strategies. The most well-known metaheuristic techniques include methods such as Genetic Algorithms, Particle Swarm Optimization, Ant Colony Algorithm, Simulated Annealing, and Artificial Bee Colony. These algorithms work on the basis of heuristic rules and randomness to reach the global best solution.

The main advantage of metaheuristic techniques is that they can effectively search a wide solution space without getting stuck in local minima. Especially in high-dimensional and multimodal problems, metaheuristics can provide quite successful results in cases where classical methods fail.

Another important aspect of these techniques is that they can be easily integrated into modern applications that require big data and high computational power thanks to their parallelizable structures. In this way, they are widely used in artificial intelligence, machine learning, and optimization-based decision support systems.

3.3. Applications of Optimization in Artificial Intelligence

Artificial intelligence systems generally require an optimization process to achieve high success in decision-making, learning, and prediction processes. Optimization has become one of the fundamental building blocks of artificial intelligence and has taken its place as an indispensable tool in a wide variety of fields.

In the field of Machine Learning, optimization is used to determine the most appropriate weights during model training. For example, minimizing the error function in artificial neural networks is the basis of the learning process. In this context, techniques such as gradient descent are directly based on optimization.

In Natural Language Processing (NLP) applications, many processes are guided by optimization algorithms, from setting the parameters of language models to generating meaningful sentences. In translation systems, finding the most appropriate word order is also an optimization problem.

In Computer Vision applications, processes such as object recognition, image segmentation and feature selection are improved through optimization. Obtaining the most accurate model from visual data usually requires optimizing a multi-dimensional and complex solution space.

Artificial Intelligence-Aided Decision Systems is another area where optimization is used intensively. Optimization algorithms provide the most appropriate decisions in problems such as production planning, resource allocation, logistics and financial portfolio management.

In addition, in areas such as reinforcement learning, agents interacting with their environments to develop strategies that will yield the highest rewards are directly dependent on optimization processes. The agent tries to maximize a value function by evaluating the consequences of its actions.

As a result, optimization plays a central role in almost all sub-branches of artificial intelligence, both theoretically and practically. Correct modeling of problems and selection of appropriate optimization techniques are one of the most critical steps that directly affect the success rate of systems.

4. CONCLUSION

Mathematical optimization is an indispensable component of the success of artificial intelligence systems. This field covers the entirety of mathematical methods that aim to find the best (minimum or maximum) value of a target function. Optimization in artificial intelligence systems forms the basis of many basic functions, from the learning process to decision-making mechanisms. For example, in machine learning, training of model parameters is usually based on minimizing the loss function. In this context, techniques such as gradient descent and stochastic gradient descent (SGD) are frequently used.

Both classical and modern techniques offer critical solutions for different problem types and application areas. While classical optimization methods include techniques such as linear programming, quadratic programming and integer programming; modern approaches include heuristic and meta-heuristic methods such as evolutionary algorithms, particle swarm optimization, genetic algorithms, swarm intelligence and differential evolution. In high-dimensional and complex systems such as deep learning, non-convex optimization problems are at the forefront. Special algorithms and learning strategies developed for such problems both increase computational efficiency and improve model performance.

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