ARCENG
INTERNATIONAL JOURNAL OF ARCHITECTURE AND ENGINEERING

ISSN
2822-6895

Article Arrival Date
13.04.2025

Article Type
Research Article

Article Published Date
20.06.2025

MATHEMATICAL APPROACHES ON FOURIER-BASED APPLICATIONS IN ARTIFICAL INTELLIGENCE

BEKİR DANIŞ [1]

[1] Dr., Aydın Adnan Menderes University, Digital Transformation Office, https://orcid.org/0000-0002-3532-2670,

Abstract

Fourier analysis enables us to convert the domain of functions into frequency domain. Thus, it is helpful tool to figure out and interpret the big data. Furthermore, Fourier based applications in artifical intelligence are very significant in terms of analyzing the data and developing effective models. In this research, we focus on the connection between Fourier analysis and artifical intelligence and this paper includes the mathematical background of Fourier analysis and their application to artifical intelligence such as neural operators, spectral learning. This study links usual harmonic analysis with data-driven intelligence.

Keywords: Artifical intelligence, Fourier analysis, mathematics

1. INTRODUCTION

Fourier analysis is named after the French mathematician Jean-Baptiste Joseph Fourier and allows a function, especially complex signals, to be expressed as the sum of simpler functions (usually sine and cosine functions). This analysis is widely used in many fields such as signal processing, communications, image processing, and audio engineering. Artificial intelligence, on the other hand, is a field that develops various algorithms to learn from data and obtain meaningful outputs. Fourier analysis plays a critical role in artificial intelligence applications, especially in areas such as data preprocessing, feature engineering, signal processing, and time series analysis. The relationship between artificial intelligence and Fourier analysis is based on the extraction of meaningful features from large data sets, especially in deep learning and machine learning algorithms. The Fourier transform analyzes complex data sets and reveals important frequency components, which allows artificial intelligence algorithms to make more accurate predictions. For further details and investigations, take the resources [1-5] into consideration.

Fourier analysis is a method of breaking down a function, usually into simpler trigonometric components (sine and cosine). Artificial intelligence (AI) is a field that allows machines to learn from data and derive meaningful results from that data. Fourier analysis is an important tool, especially in areas such as signal processing, image processing, audio analysis, and time series data. In this article, we will examine the uses of Fourier analysis in artificial intelligence and machine learning, and discuss how it plays a role, especially in areas such as deep learning and signal processing. We will emphasize the importance of Fourier transformation in data analysis, model training, and optimization processes.

2. FOURIER SERIES AND TRANSFORMATION

Let $f(x)$ be a 2π-periodic, integrable function. Then, its Fourier series given as follows:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}, \quad c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$$

This representation allows the function to be separated into independent frequency components.

For $f \in L^1(\mathrm{R})$, the Fourier Transform is introduced by:

$$\mathcal{F}[f](\omega) = \hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

After these basic definitions, it is obviously obtained that the inverse Fourier transform reconstructs the original signal:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t} d\omega$$

3. PARSEVAL'S, PLANCHEREL AND CONVOLUTION THEOREMS

In this section, we state major theorems which play a central role in applied analysis. Parseval's theorem for Fourier series is that:

$$\int_{-\pi}^{\pi} |f(x)|^2 dx = 2\pi \sum_{n=-\infty}^{\infty} |c_n|^2$$

In a similar manner, we give Plancherel's theorem for the Fourier transform as follows:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{f}(\omega)|^2 d\omega$$

Then, we define the convolution: for functions f and $g \in L^1(\mathrm{R})$, the Fourier transform of their convolution is:

$\mathrm{F}[f * g](\omega) = \mathrm{F}[f](\omega) \cdot \mathrm{F}[g](\omega)$

This equality play an important role in optimizing convolution operations with the help of Fast Fourier Transforms (FFT).

4. MATHEMATICAL MODELS IN DEEP LEARNING

4.1. CONVOLUTIONAL NEURAL NETWORKS

Thanks to the convolution theorem, CNNs can be accelerated using FFT. This reduces the computational complexity from $O(n^2)$ to $O(n \log n)$, thus allowing for more efficient processing of high-dimensional data, such as images and videos.

4.2. POSITIONAL ENCODING

Positional encodings, such as:

$\gamma(x) = [\sin(2\pi Bx), \cos(2\pi Bx)]$

enables networks to generalize in both spatial and temporal domains and provides a powerful tool in transformer-based models and other sequential data processing systems.

4.3. SPECTRAL BIAS

In neural networks, there is a tendency to first learn low-frequency components. Mathematically, if $f(x) = \sum \hat{f}(\omega) e^{i\omega x}$, the model tends to prioritize:

$$|\hat{f}_{\text{learned}}(\omega)| \gg 0 \quad \text{for small } \omega$$

This is called spectral bias, and the model's learning process tends to adapt to low frequency datas.

ISSN 2822-6895

5. FOURIER NEURAL OPERATORS

Fourier Neural Operators (FNOs) are a class of models developed to solve parametric partial differential equations (PDEs) in an efficient and scalable manner. These models operate in the Fourier domain to learn relationships that map the initial states of the system to the final states.

Given input $u(x)$, FNOs perform:

$$v = \mathrm{F}(u), \qquad w = R_\theta(v), \qquad \hat{u} = \mathrm{F}^{-1}(w)$$

where R_θ is a learnable function in the frequency space, making the model robust to changes in resolution and grid size.

FNOs provide significant advantages over traditional methods in PDE solutions, especially when working with systems with complex geometries or mesh structures. Thanks to their strong generalization capability across different resolutions, they greatly reduce the need for retraining for each new mesh or grid structure.

6. SIGNAL PROCESSING AND FOURIER TRANSFORMATION

Signal processing is a fundamental method widely used in many fields, from electrical engineering to biomedical engineering. Fourier analysis enables the analysis of the fundamental components of the signal by transforming complex signals in the time domain to the frequency domain. Artificial intelligence can produce more effective and efficient results by utilizing these signal processing techniques. In particular, Fourier transform plays a critical role in reducing noise in signals, extracting features, and converting data into a more appropriate format.

7. IMAGE PROCESSING AND FOURIER ANALYSIS

It is widely used in many fields such as image processing, computer vision and image analysis. The Fourier transform converts an image into the frequency domain, allowing the examination of certain frequency components. In this way, edges, details and patterns in the image can be identified more effectively. Artificial intelligence algorithms can extract meaningful features from the image using the Fourier transform and use these features in tasks such as classification, segmentation or prediction.

8. LANGUAGE AND VOICE PROCESSING

Audio signals and natural language processing are among the prominent application areas of Fourier analysis. The Fourier transform can be used effectively in tasks such as speech recognition, voice command systems, and music analysis by separating audio signals into frequency components. Artificial intelligence can perform more accurate classifications by analyzing meaningful frequency components in sound through this transformation. For example, a voice assistant processes the incoming audio signal with Fourier analysis, evaluates the frequency components of the human voice, and produces appropriate responses.

9. TIME SERIES AND FOURIER TRANSFORM

Time series analysis is a method frequently used in the analysis of large data sets in finance, healthcare and many industries. The Fourier transform converts time series data into the frequency domain, allowing cyclical behaviors and certain patterns to be revealed. With this transformation, AI can detect significant frequency components in the time series and use this information to produce more accurate predictions.

ISSN
2822-6895

10. CONCLUSION

Fourier analysis provides a powerful and mathematically sound foundation for developing AI models. Thanks to the advantages offered by the frequency domain, AI models can achieve higher efficiency, better interpretability, and increased robustness. With ongoing research, the combination of classical harmonic analysis and modern machine learning techniques will likely lead to more advanced and scalable model structures.

Fourier analysis stands out as a fundamental mathematical tool in the field of artificial intelligence and plays a critical role in many areas such as signal processing, image analysis, audio data, time series analysis, and data preprocessing. By transforming complex data into the frequency domain, Fourier transform reveals the prominent components of the data more clearly. In this way, the accuracy of artificial intelligence and machine learning algorithms increases, model training processes are accelerated, and overall efficiency increases. Therefore, the relationship between Fourier analysis and artificial intelligence is of great importance both theoretically and practically and is expected to pave the way for more research and innovation in the coming years.

11. REFERENCES

[1] Bronstein M., Geometric Deep Learning: Going Beyond Euclidean Data, IEEE Signal Processing Magazine, 2017.

[2] Li Z., Kovachki N., Azizzadenesheli K., Fourier Neural Operator for Parametric PDEs, arXiv:2010.08895, 2020.

[3] Mallat S., *A Wavelet Tour of Signal Processing*, Academic Press, 1999.

[4] Tancik M. et al., Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains, NeurIPS, 2020.

[5] Xu Z., Zhang Y., Wang Y., Frequency Principle: Fourier Analysis Sheds Light on Deep Neural Networks, arXiv:1901.06523, 2019.